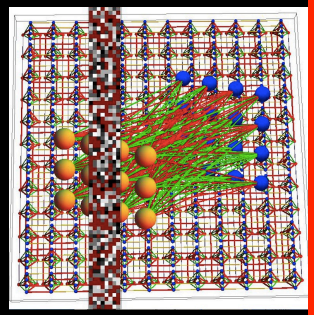
Adiabatic Quantum Computers: Testing and Selected Applications

- Quantum Dragon Devices (FETs)
- Adiabatic Quantum Computers
  - Finite-T Decoherence
  - AQC + SW
  - Testing D-Wave AQCs
  - Boltzmann machines + AQC



Mark A. Novotny

Supported in part by NSF and PNNL

MISSISSIPPI STATE UNIVERSITY

NOTES
REDACTED VERSION

Some of the results presented have yet to be published. Therefore, in order to not have any difficulties with journal copyright or intellectual property issues certain slides will only have the words 'REDACTED SLIDE'. These redactions will remind attendees of what was covered in the order of the slides.

In addition, in order to not have copyright issues with other publications, figures from other authors also will have 'REDACTED FIGURE' written in their place.

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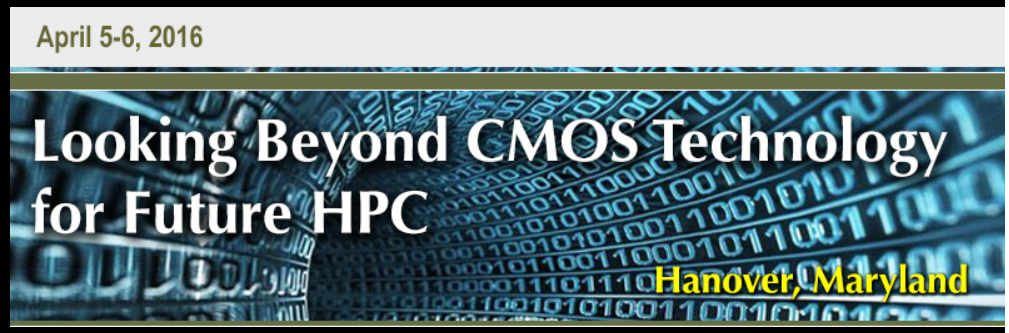


Who has done computations …

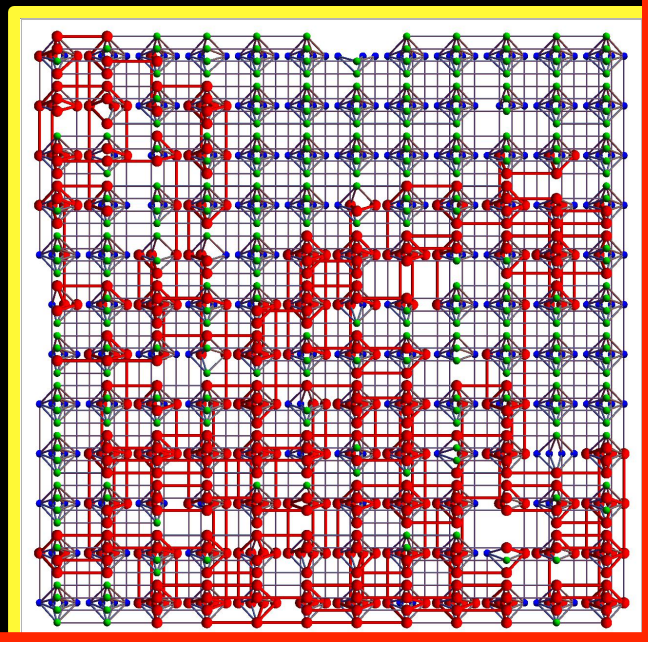
Computer CPU based on vacuum tubes?

REDACTED FIGURE

REDACTED FIGURE

Data storage on paper tape?

RAM from coil-wound solenoids?



Programmed in Assembly language?

A 1000+ qubit quantum computer?

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

In 35+ years, ***two*** disruptive computing innovations:

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

In 35+ years, ***two*** disruptive computing innovations:

Gordon Moore
1965

REDACTED
FIGURE

Disruptive to computing

Been doing computational physics since
PhD in Physics from Stanford University in 1978

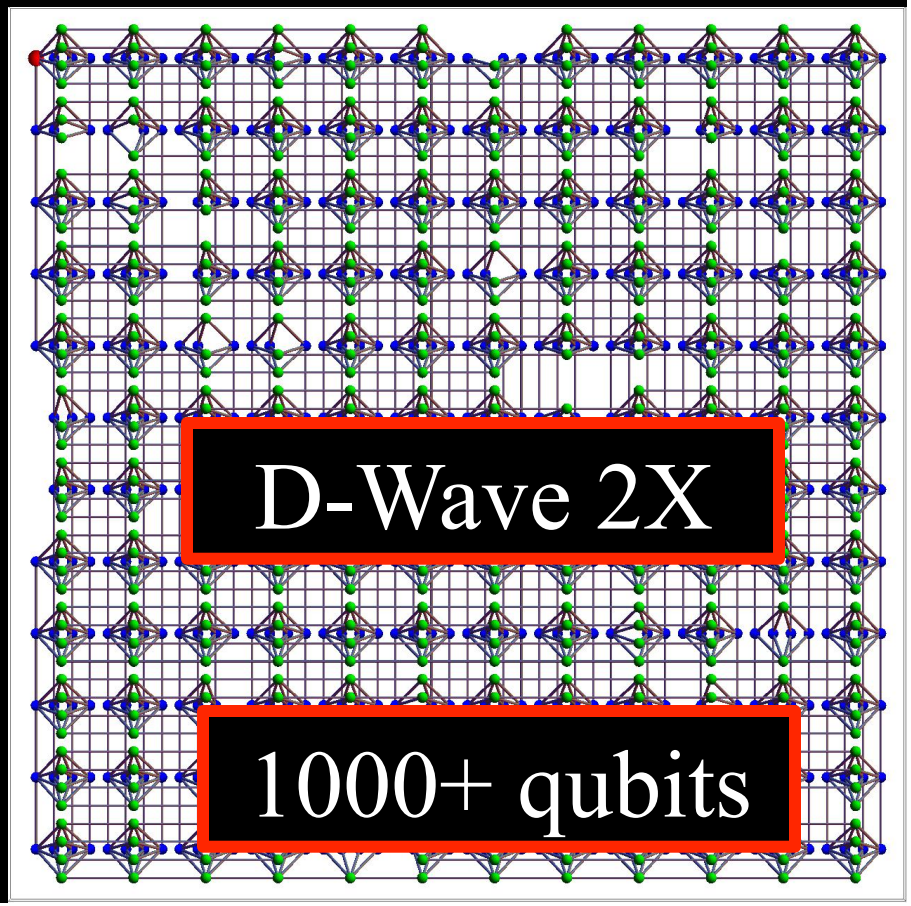
In 35+ years, ***two*** disruptive computing innovations:

Quantum Computing

"Perhaps the quantum computer will change our everyday lives in this century in the same radical way as the classical computer did in the last century."

Nobel press release (Oct. 2012)

David J. Wineland
Serge Haroche



Quantum Dragons (FETs)

Quantum Transport Experiment
(Ballistic electron motion: zero electrical resistance)

REDACTED FIGURE

Four-terminal resistance of a ballistic quantum wire

R. de Picciotto*, H. L. Stormer*†, L. N. Pfeiffer*, K. W. Baldwin* & K. W. West*

* Bell-Labs, Lucent Technologies, Murray Hill, New Jersey 07974, USA
† Departments of Physics and Applied Physics, Columbia University,

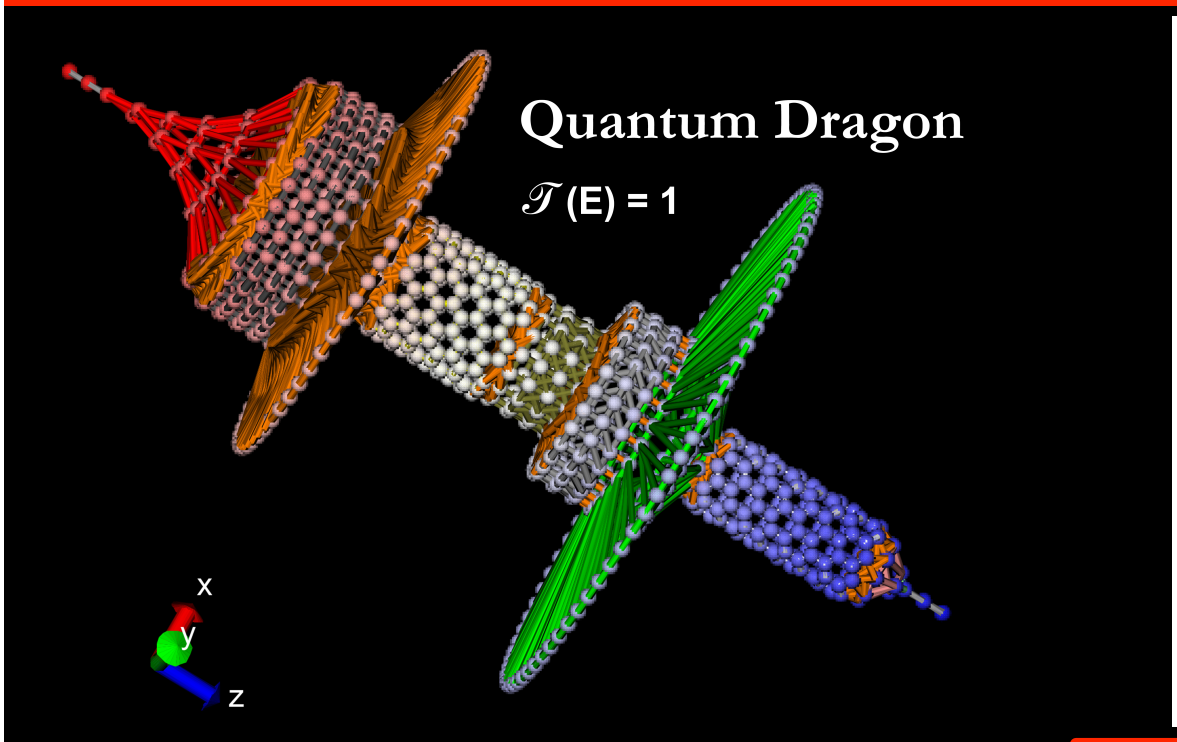
NATURE | VOL 411 | 3 MAY 2001 |

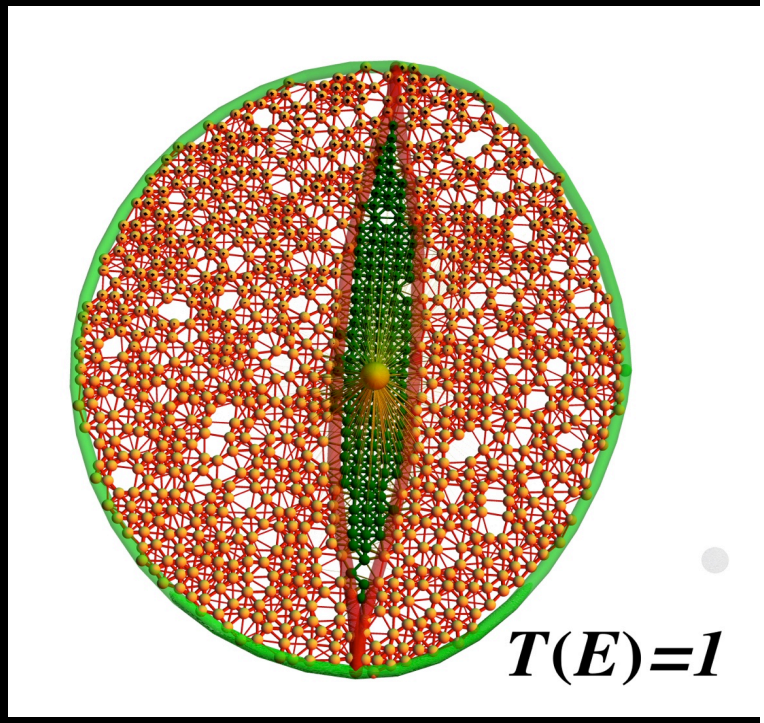
Total transmission: $\mathcal{T}(E)=1$

Four probe: $G=\infty$

Two probe: $G=G_0=2\ e^2/h$

Quantum Dragons
$\mathcal{T}(E)=1$ but *strong disorder*





Find exact solution of Schrödinger equation (with ∞ leads)

PRB 2014
arXiv: 1502.07814
Patent Pending

Quantum Dragon FETs
$\mathcal{T}(E)=1$ but *strong scattering*

- Fixed constant electric field on slices
- Use Perron-Frobenius theorem (strongly connected graph)
- ***UNIQUE*** connection to leads required for a quantum dragon

REDACTED SLIDE

$\mathcal{T}(E)=1$

Quantum Dragon FETs
$T(E)=1$ but *strong scattering*

REDACTED SLIDE

US Patent Pending

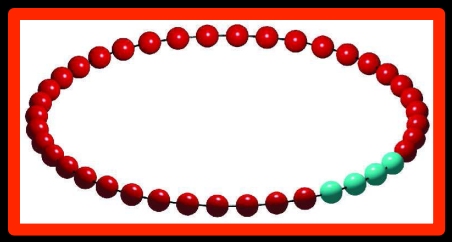
Quantum Decoherence at finite Temperature

Finite *T* Decoherence

Perturbation Theory +
Large-scale real-time quantum calculations

$$\sigma(t) = \sqrt{\sum_{i=1}^{D_S-1} \sum_{j=i+1}^{D_S} \left|\hat{\rho}_{ij}(t)\right|^2}$$

$$\begin{aligned}
\mathscr{E}\left(\sigma^2\right) &= \frac{1}{2} e^{-2\beta\left(F_E(2\beta)-F_E(\beta)\right)}\left(1-e^{-2\beta\left(F_S(2\beta)-F_S(\beta)\right)}\right) \\
&- \frac{2D}{D+1} e^{-3\beta\left(F_E(3\beta)-F_E(\beta)\right)} \\
&\times \left(e^{-2\beta\left(F_S(2\beta)-F_S(\beta)\right)} - e^{-3\beta\left(F_S(3\beta)-F_S(\beta)\right)}\right) \\
&+ \frac{3D}{2(D+1)} e^{-4\beta\left(F_E(2\beta)-F_E(\beta)\right)} e^{-2\beta\left(F_S(2\beta)-F_S(\beta)\right)} \\
&\times \left(1-e^{-2\beta\left(F_S(2\beta)-F_S(\beta)\right)}\right) .
\end{aligned}$$



$$\lim_{\beta\to\infty} \mathcal{E}\left(\sigma^2\right) \approx \frac{(g_S-1)(g_S g_E-1)}{2 g_S^2 g_E^2}$$

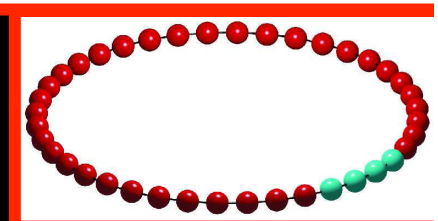
$$\lim_{\beta\to 0} \mathcal{E}\left(\sigma^2\right) = \frac{D_S-1}{D_S D_E+1} + \cdots$$

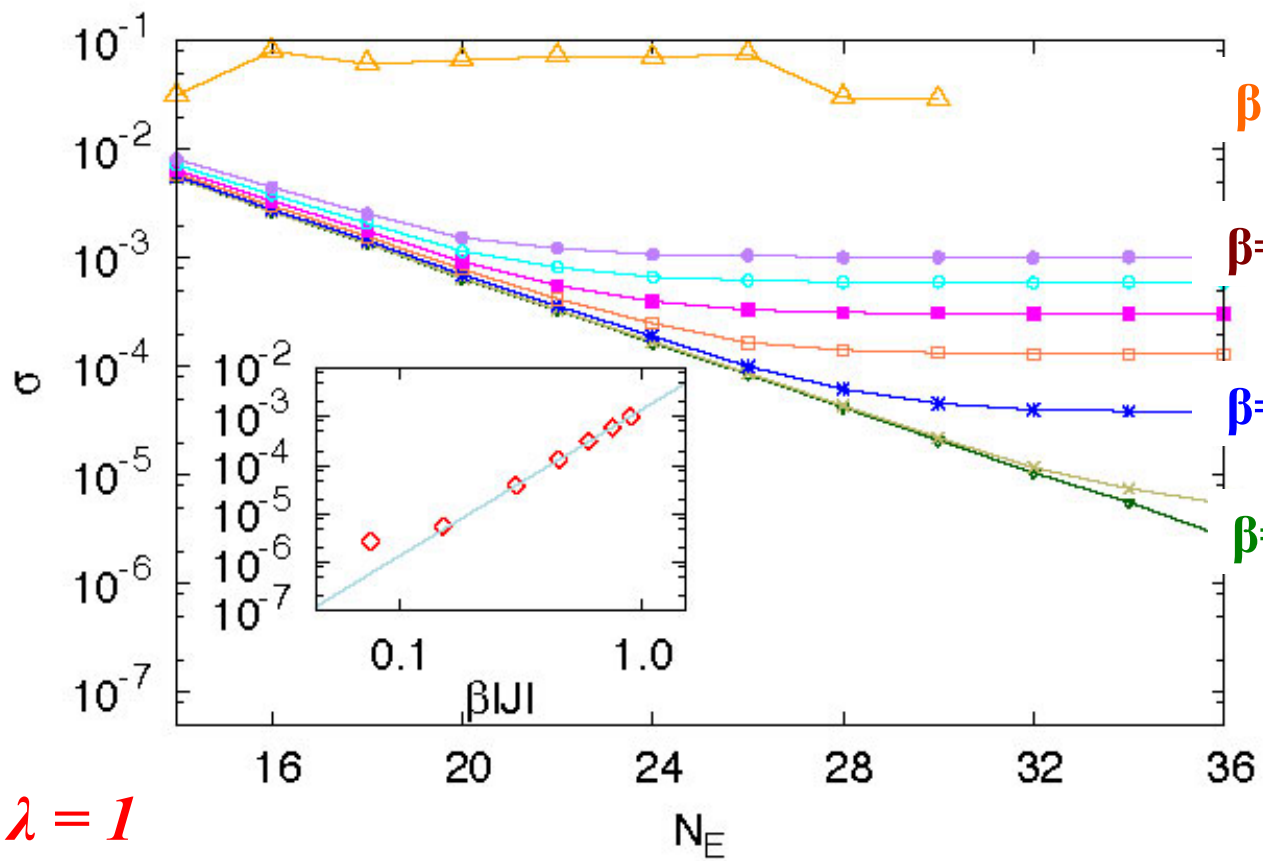
Phys Rev A **93**, 032110 [46 pages] (2016)

Finite T Decoherence
Finite σ if $\lambda \neq 0$ or $T < \infty$

$$\sigma(t) = \sqrt{\sum_{i=1}^{D_S-1} \sum_{j=i+1}^{D_S} |\hat{\rho}_{ij}(t)|^2}$$

$$\mathcal{H} = \mathcal{H}_S + \mathcal{H}_E + \lambda \mathcal{H}_{SE}$$



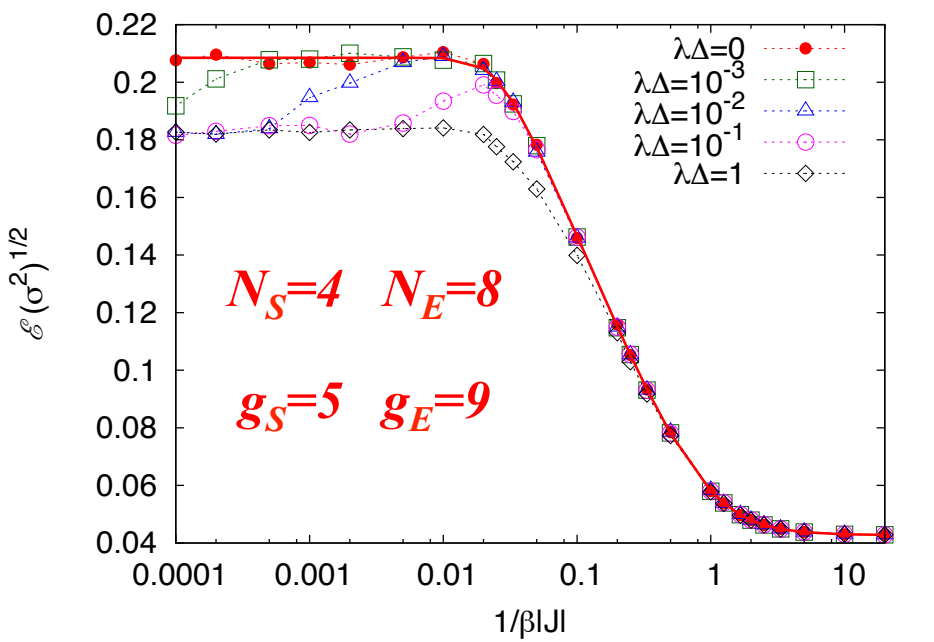


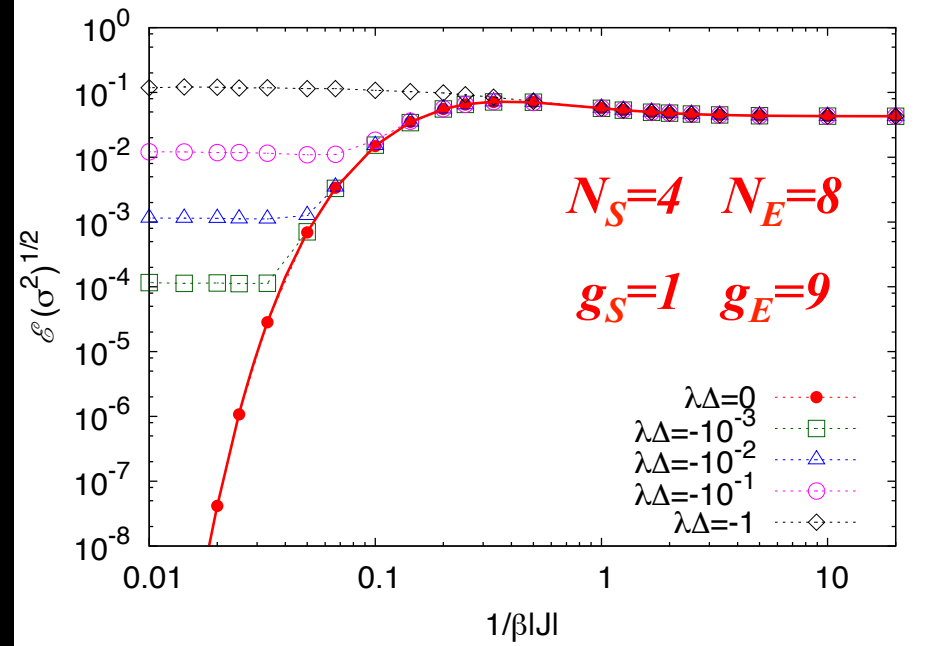
$\lambda = 1$

PRA (2016)

Finite *T* Decoherence

Perturbation Theory + Simulations (*NO* adjustable parameters)





$$\sigma(t) = \sqrt{\sum_{i=1}^{D_S-1} \sum_{j=i+1}^{D_S} \left|\hat{\rho}_{ij}(t)\right|^2}$$

$$\lim_{\beta\to\infty} \mathcal{E}\left(\sigma^2\right) \approx \frac{(g_S-1)(g_S g_E-1)}{2 g_S^2 g_E^2}$$

$$\begin{aligned}\mathcal{E}\left(\sigma^2\right) &= \frac{1}{2} e^{-2\beta(F_E(2\beta)-F_E(\beta))}\left(1-e^{-2\beta(F_S(2\beta)-F_S(\beta))}\right) \\ &-\frac{2D}{D+1} e^{-3\beta(F_E(3\beta)-F_E(\beta))} \\ &\times\left(e^{-2\beta(F_S(2\beta)-F_S(\beta))}-e^{-3\beta(F_S(3\beta)-F_S(\beta))}\right) \\ &+\frac{3D}{2(D+1)} e^{-4\beta(F_E(2\beta)-F_E(\beta))} e^{-2\beta(F_S(2\beta)-F_S(\beta))} \\ &\times\left(1-e^{-2\beta(F_S(2\beta)-F_S(\beta))}\right).\end{aligned}$$

Phys Rev A **93**, 032110 [46 pages] (2016)

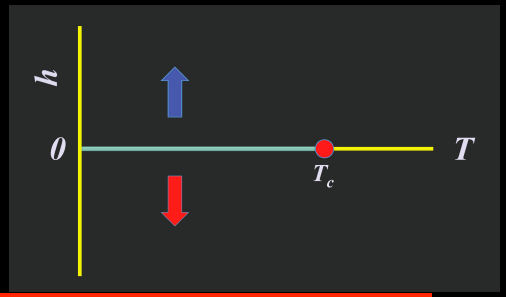
Adiabatic Quantum Computer (AQC)

Small-World Connections

Ising Ferromagnet or Spin Glass

$y_T=1/\nu$ *critical exponents depend on dimension d*

$$\mathcal{H}_0 = -\sum_{\langle i,j\rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i^z\sigma_i^z$$

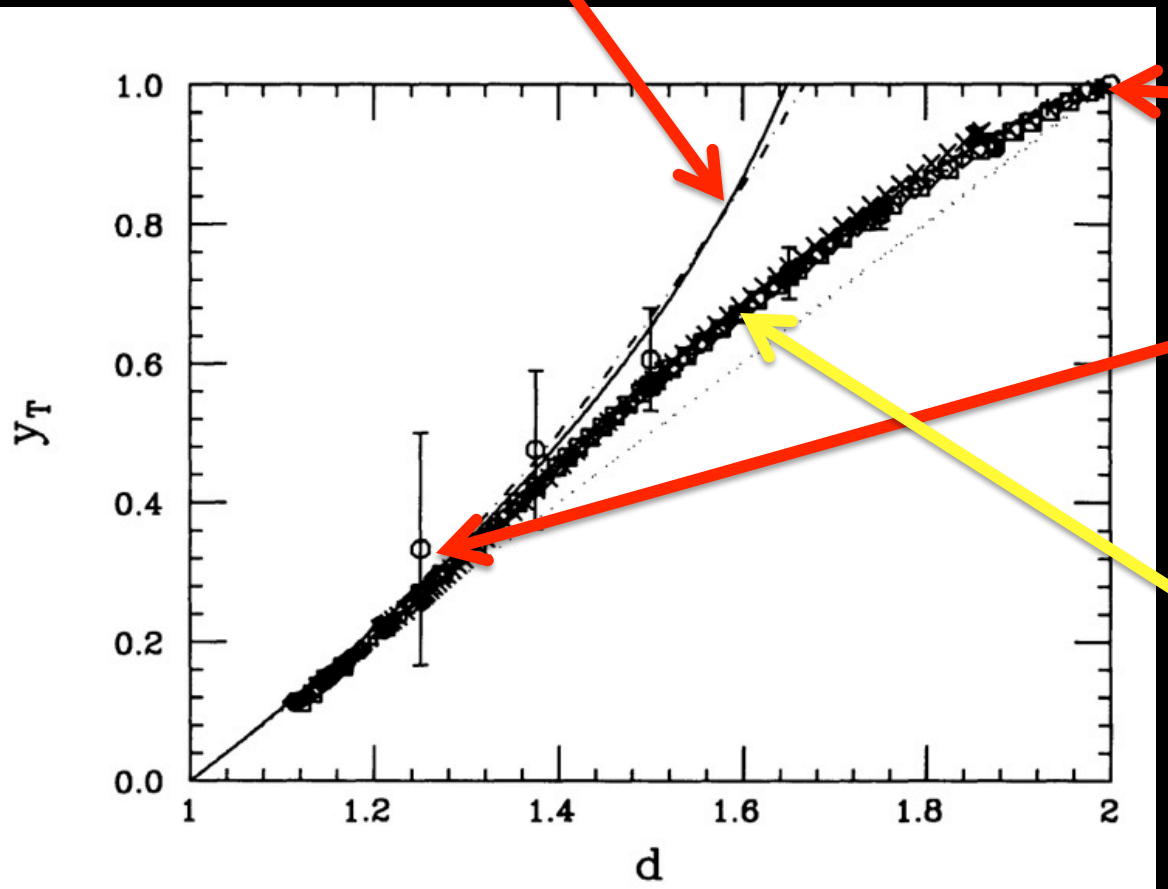


$d=1+\varepsilon$ expansions

$\nu=1$ exact $d=2$

resummed $d=4-\varepsilon$ expansions

Numerical transfer matrix

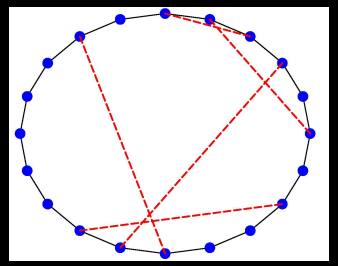


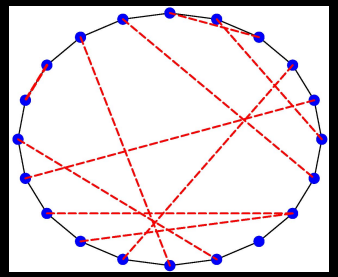
Ferromagnet

Novotny *PRB 1992*

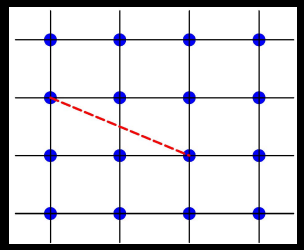
Intro. to Small World Networks

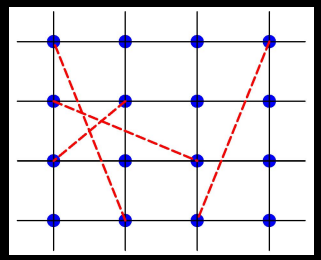
Construct Small-World Networks

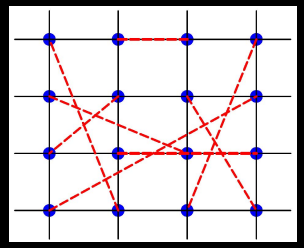




d*=1 to *SW
or
d*=2 to *SW



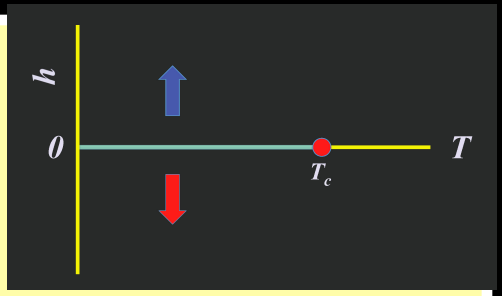




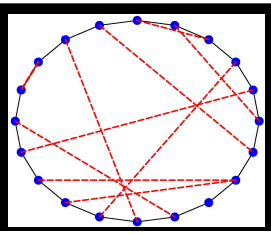
- ***p*** = Fraction of bonds added
- ***L*** (or L^2) *lattice nodes*
- Average distance between nodes ***d~ln(L)***

Small World (*z*-model)

- Start with *d=1* chain of spins
- Randomly connect pairs of spins
- All spins have *z* interactions (strength 1)

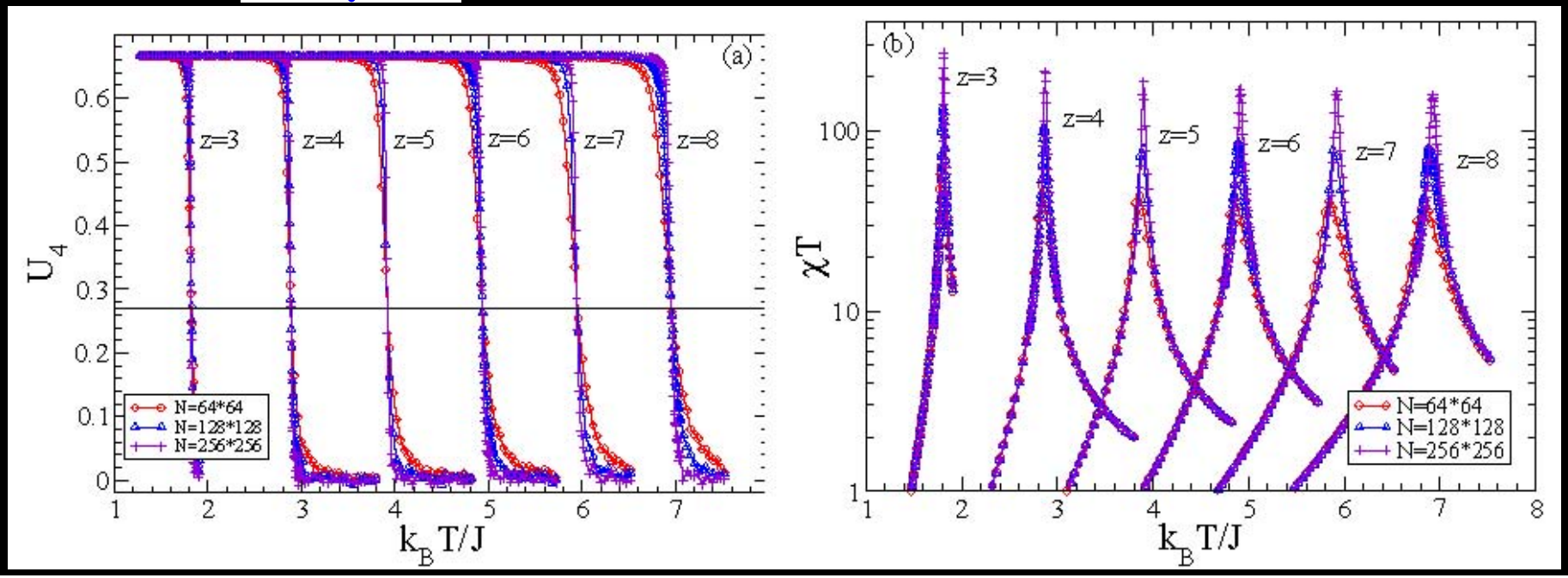


$z=3$



Ising Ferromagnet

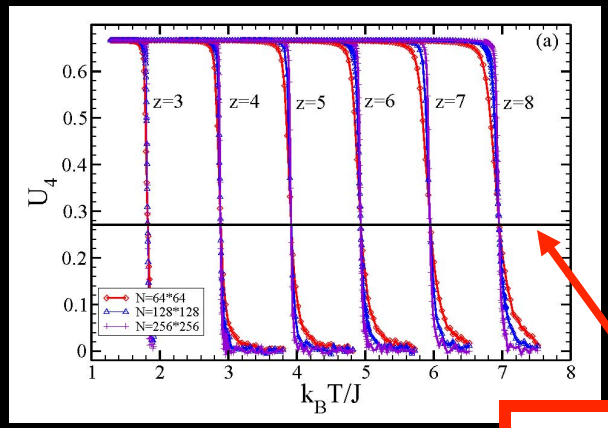
Introduced by Scalettar 1991



Small-world (z-model) y_T mean field

➢ Binder 4th-order cumulant

Ising Ferromagnet



$$t = |(T - T_c)/T_c|$$

$$U_4(t) = f_U\left(tN^{\frac{1}{2}}\right)$$

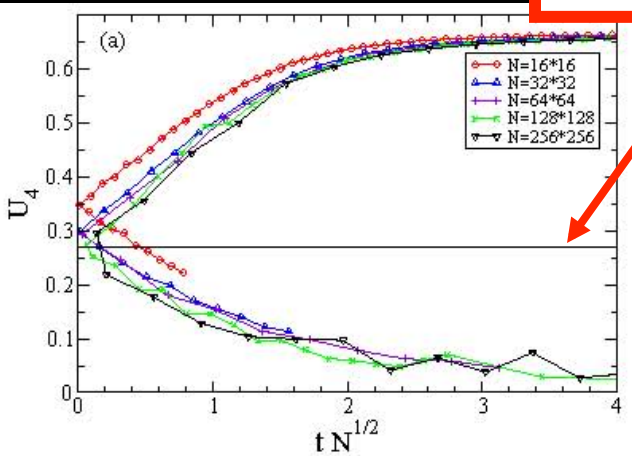
$$\frac{\partial U_4(t)}{\partial t} = N^{\frac{1}{2}} f'_U\left(tN^{\frac{1}{2}}\right)$$

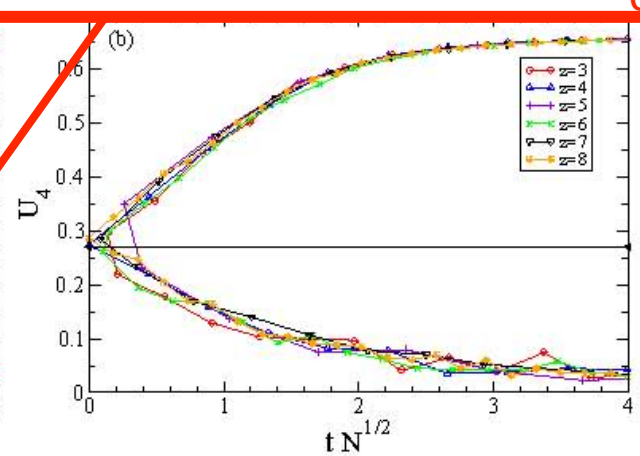
Brézin

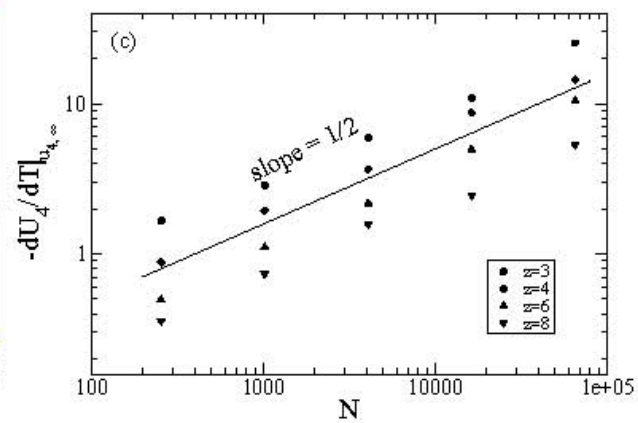
Zinn-Justin

1985

Exact N=∞ value at T_c







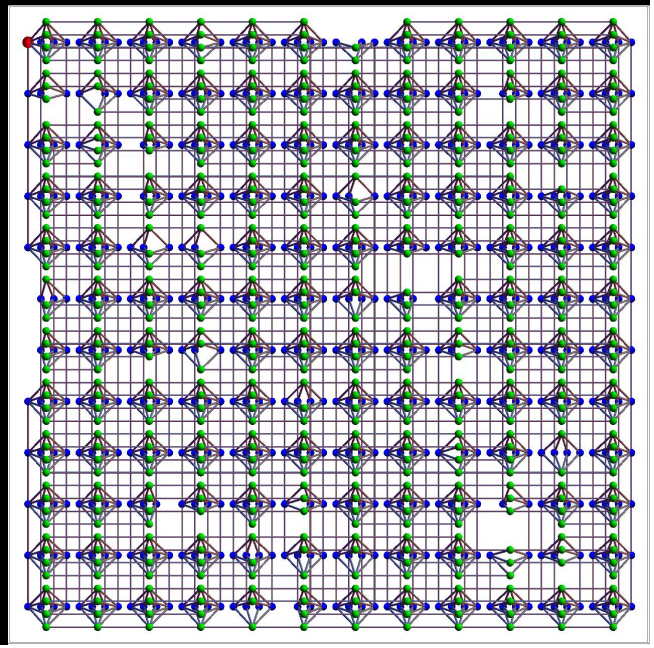
D-Wave: improve the AQC?

D-Wave 2X with 1000+ qubits does …

D-Wave 2X is NOT ideal adiabatic quantum computer

No finite-temperature spin-glass transition
Chimera $K_{4,4}$ lattice

Katzgraber, et al 2014



D-Wave: improve the AQC?

REDACTED
SLIDE

How to improve D-Wave AQC?

REDACTED SLIDE

Better Oracle?

US Patent. + US Patent Pending

Adiabatic Quantum Computer (AQC)

How to think about AQC

AQC

REDACTED
SLIDE

AQC

You *give* AQC:

Oracle

bias h_j on each qubit

coupling J_{ij} between qubit pairs

You *get* (probably)

Ground state Ising spin configuration

$$\mathcal{H}_0 = -\sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i^z\sigma_i^z$$

AQC Guarantees:

You will not always get the correct answer

SIMD *Parallelization of Ising Spin Glass*

The graph is fixed; 2^N processors

Scatter problem: Given $J_{i,j}$ and h_j

PE *1* | **PE *2*** | **PE *3*** | ● ● ● | **PE 2^N**

SIMD = **S**ingle **I**nstruction **M**ultiple **D**ata

SIMD *Parallelization of Ising Spin Glass*

The graph is fixed; 2^N processors

Scatter problem: Given $J_{i,j}$ and h_j

PE *1* | **PE *2*** | **PE *3*** | ● ● ● | **PE 2^N**

GATHER solutions: find lowest E $\{s\}$

AQC as analog SIMD computer

D-Wave machine (AQC) [aDWQC]

Maximum number processors you have parallelized over?

Novotny: Thinking Machine CM-2: $2^{16}=65{,}536$

aDWQC: NASA/Google/USRA: 1000+ qubit: $2^{1000}=10^{301}$

aDWQC 1000 qubit announcement: ***June 22, 2015***

$2^{16} = 7 \times 10^{4}$

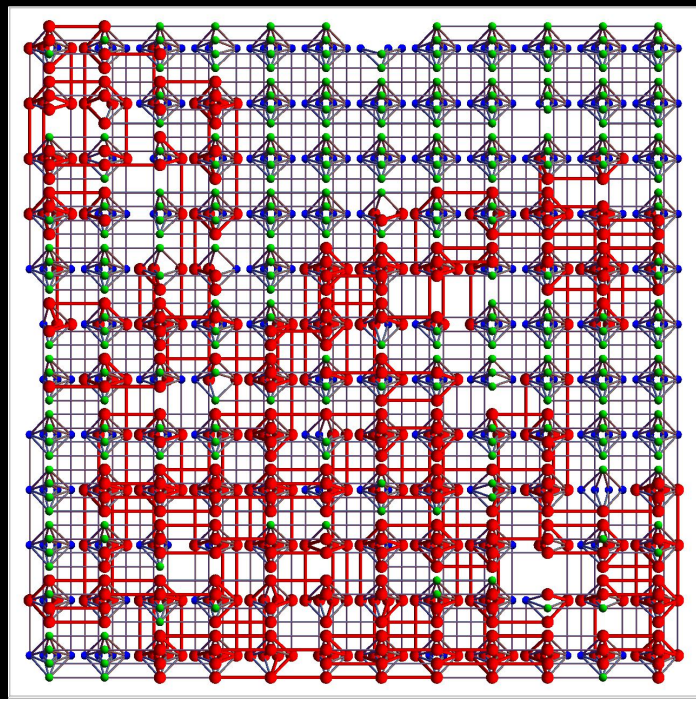
$2^{22} = 4 \times 10^{6}$

$2^{100} = 10^{29}$

$2^{230} = 10^{69}$

$2^{1000} = 10^{301}$

$2^{1097} = 10^{330}$



Adiabatic Quantum Computer (AQC)

Testing D-Wave

AQC

You *give* AQC:

Oracle

bias h_j on each qubit

coupling J_{ij} between qubit pairs

You *get* (probably)

Ground state Ising spin configuration

$$\mathcal{H}_0 = -\sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \sum_{i=1}^{N} h_i^z\sigma_i^z$$

AQC Guarantees:

You will not always get the correct answer

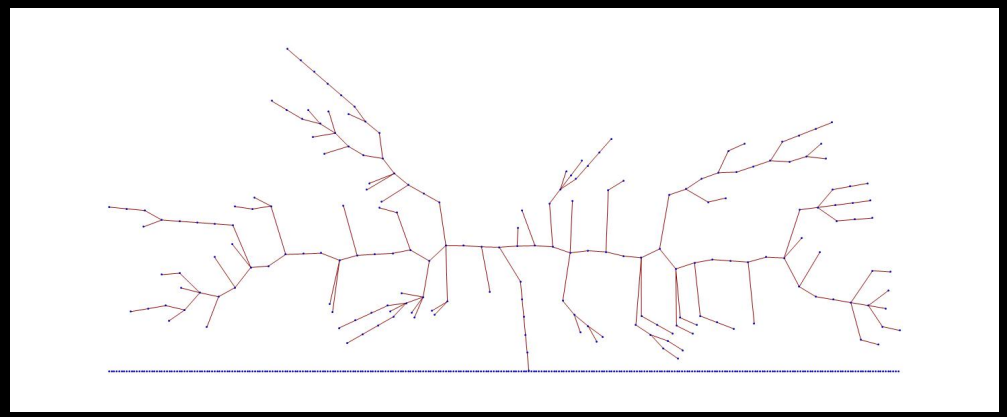
Test D-Wave

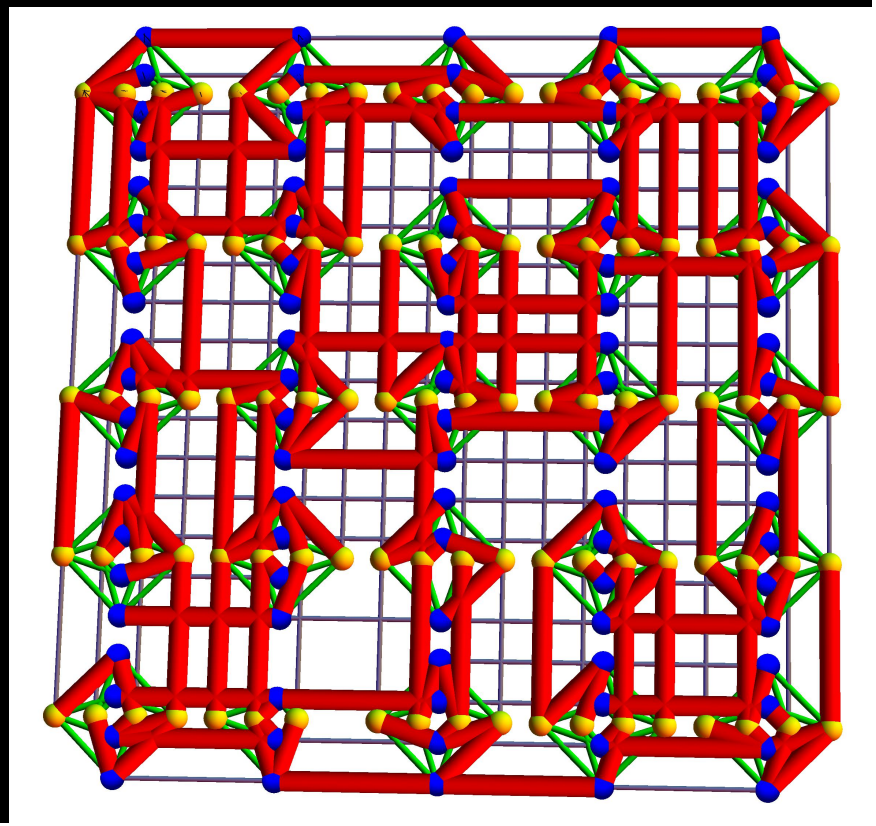
Give problems you know:

Spanning trees

(from ensemble of all spanning trees)

Oracle





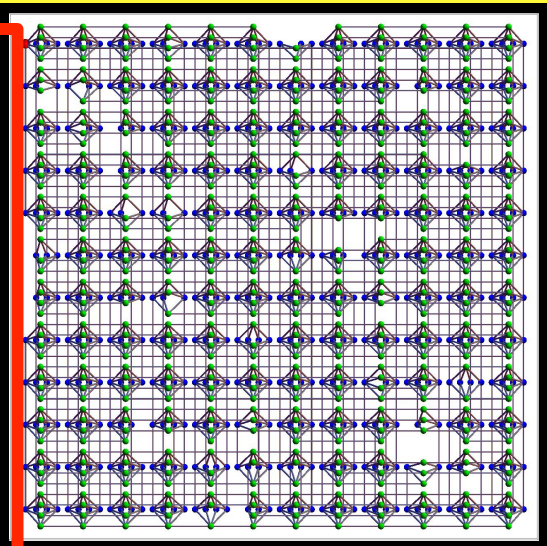
$$E_0=(N-1)\,|J|$$

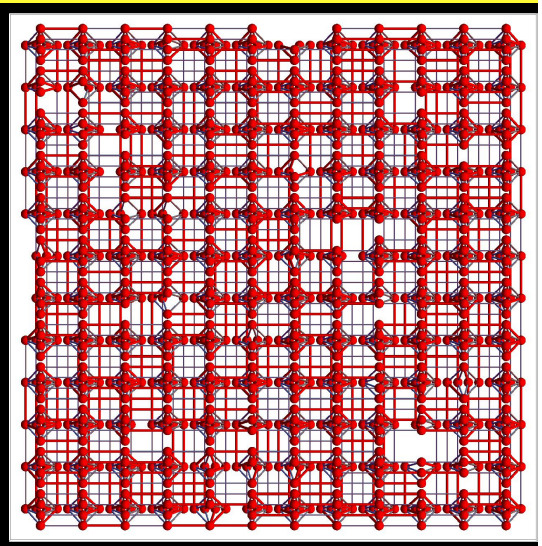
Spanning Trees: algorithm

A spanning tree uniformly drawn from the ensemble of all spanning trees

Spanning tree advantage:

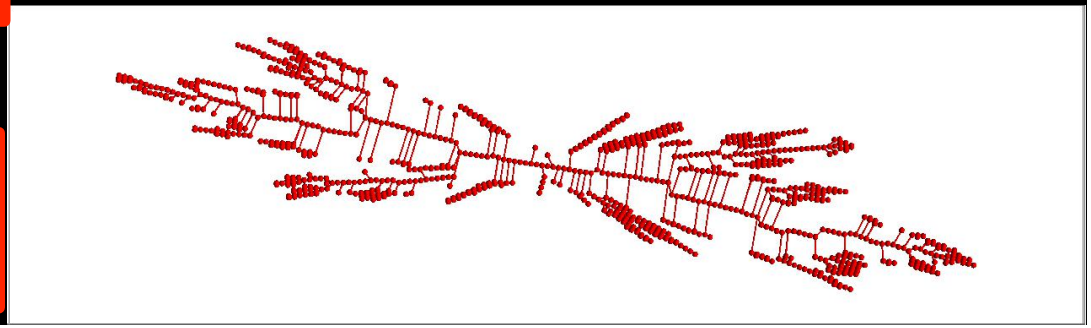
- Each tree includes all qubits
- Ensemble covers all bonds
- Ensemble is well defined
- Works on any graph
- Known groundstate
- Known spin arrangements
- Hard problem for AQC





$$J_{i,j} = \pm 1 \qquad h_j = 0$$

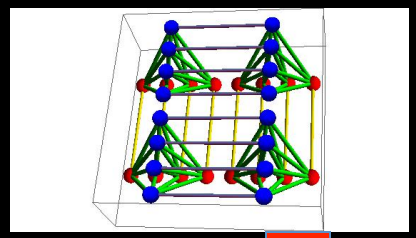
$$E_{\text{ground}} = N_{\text{qubits}} - 1$$



D-Wave 2 8×8 $K_{4,4}$ Chimera

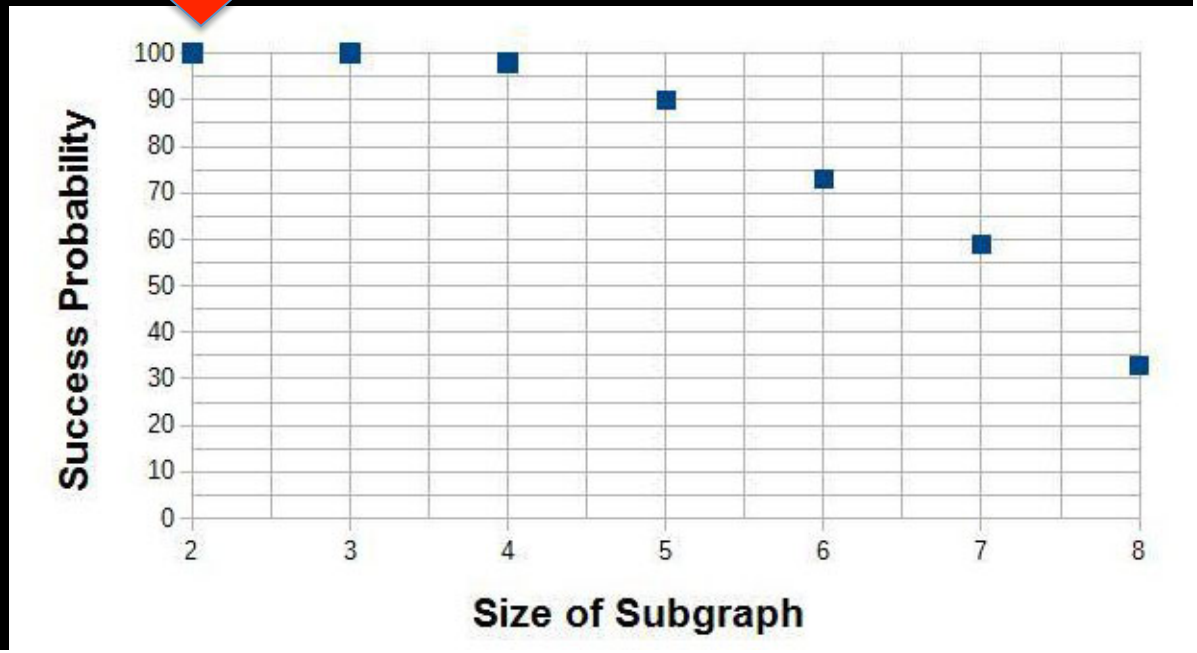
Spanning trees
496 qubit
D-Wave 2

$L \times L$ square



Good
Oracle?

100 spanning trees
up to100 submissions
each 10^3 anneals
$J_{ij} = \pm 1$



J.S. Hall, et al, Physics Procedia vol 68, 56-60 (2015)

D-Wave 2X

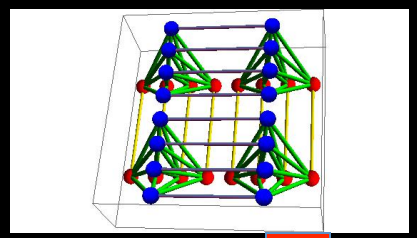
12×12 $K_{4,4}$ Chimera

Spanning trees
496 qubit
D-Wave 2

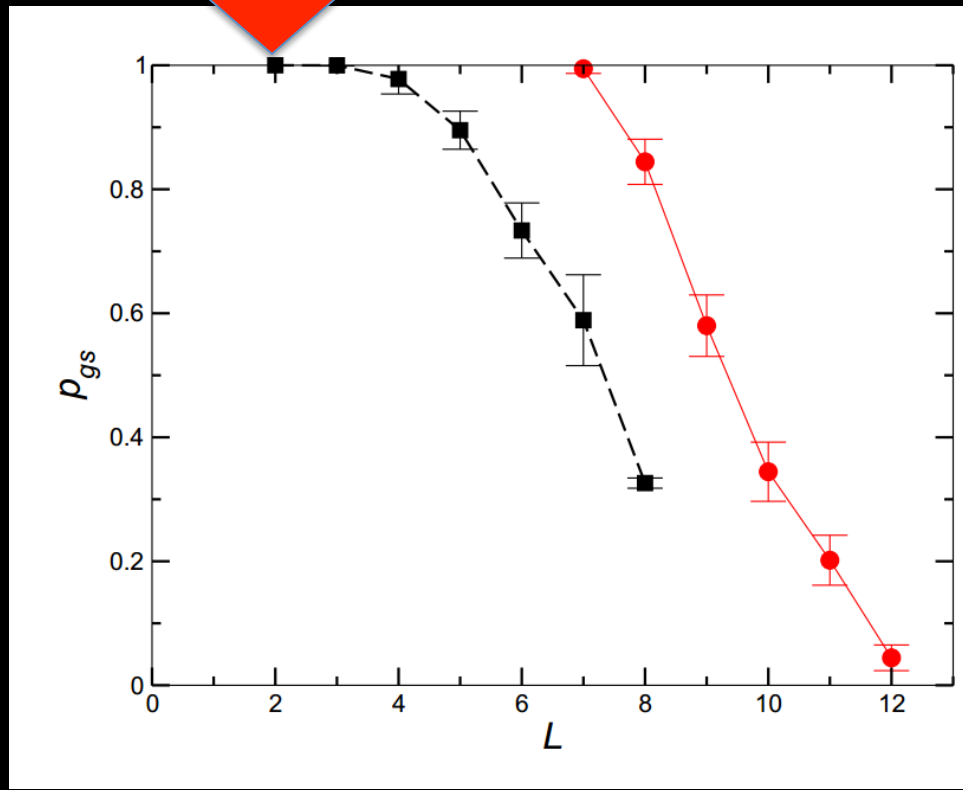
1097 qubit
D-Wave 2X

L x L square

100 spanning trees
up to100 submissions
each 10^3 anneals
$J_{ij} = \pm 1$



Good
Oracle?



M.A. Novotny, et al, J. Phys. Conf. Proc. (2016)

Test D-Wave Answer Checking

Good Oracle?

REDACTED SLIDE

D-Wave 2X: Answer Checking

REDACTED SLIDE

P_{sym} – Probability of obtaining at least a single symmetrical state as the lowest energy solution among 1000 annealing runs

D-Wave 2X: Answer Checking

REDACTED SLIDE

Hamming Distance

D-Wave 2X: Answer Checking

REDACTED SLIDE

75% of the data points are above the diagonal

Error mitigation

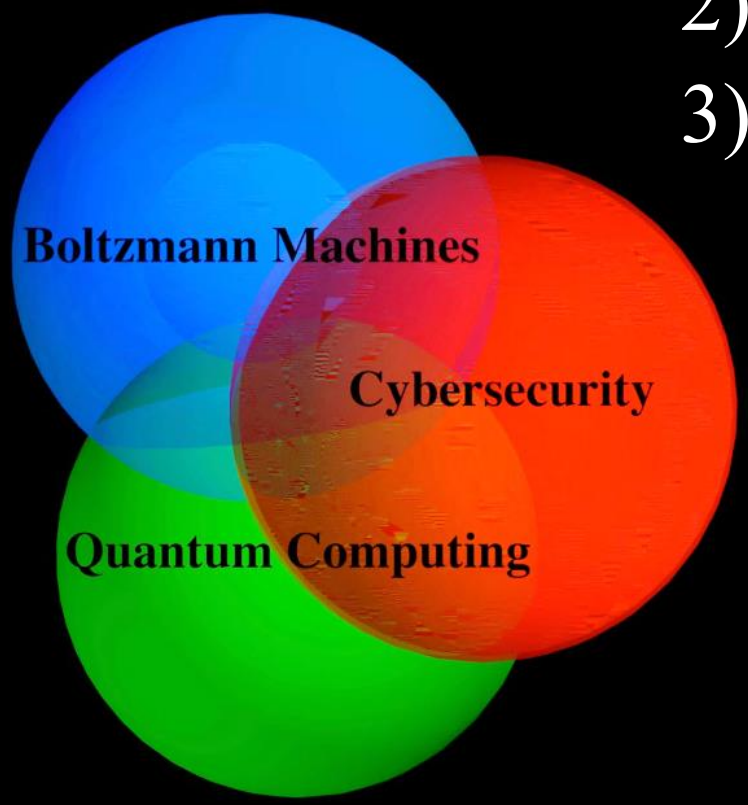
Adiabatic Quantum Computer (AQC)

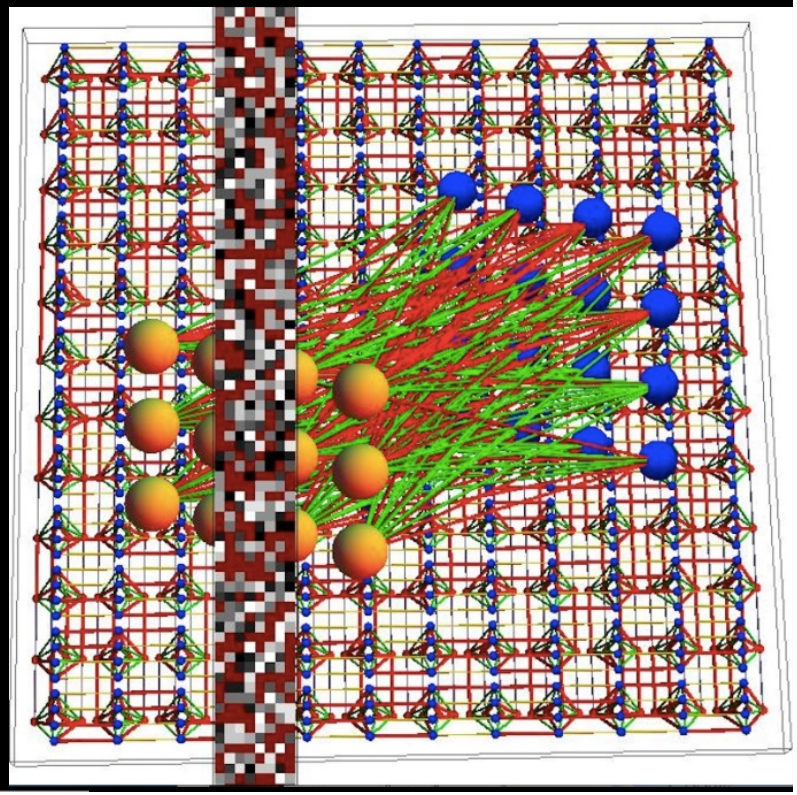
Boltzmann Machine Applications

My current application of AQC
Boltzmann Machines (Deep Belief)

Intersection of 3 fields:

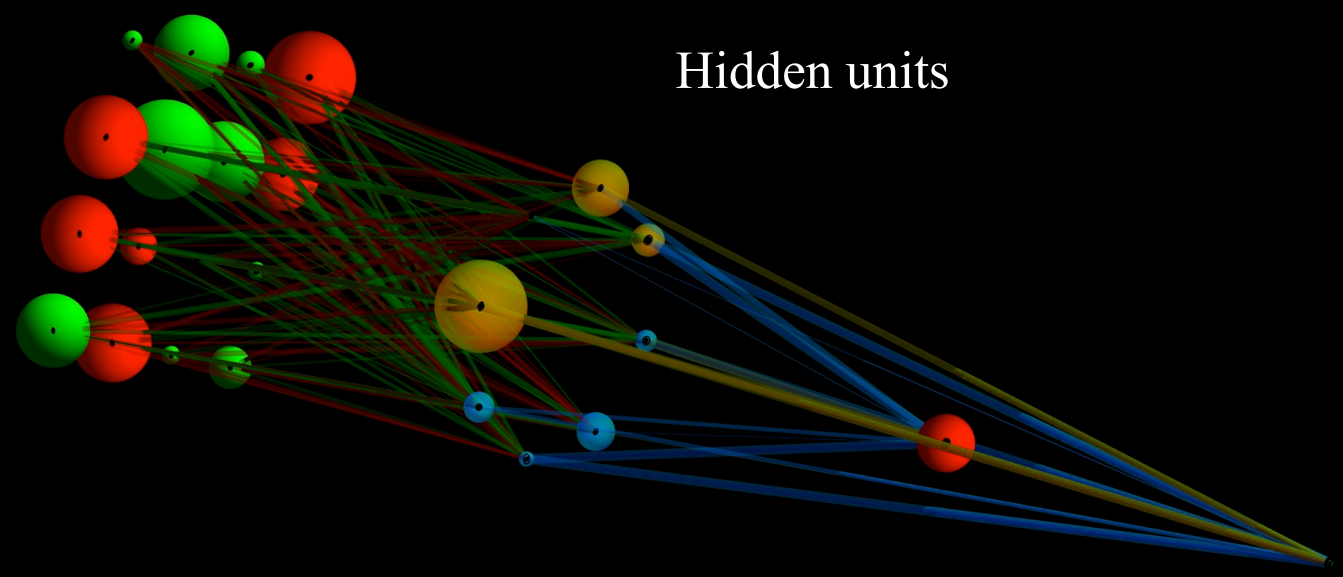
1) Cybersecurity
2) Boltzmann machines
3) Quantum computing





Boltzmann Machines

Progress: test Restricted Boltzmann Machines



Boltzmann Machines

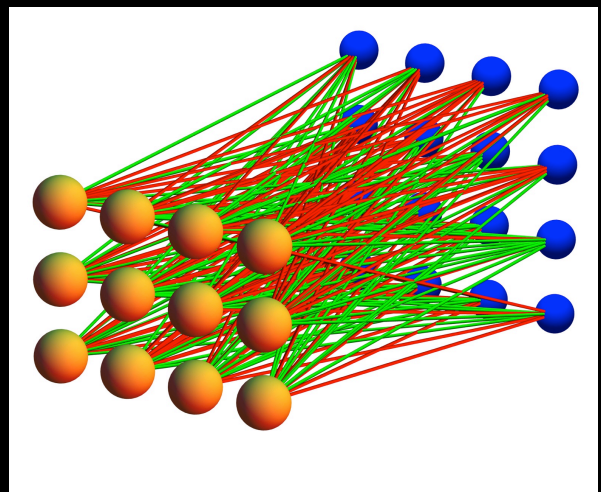
Technical Approach: Cybersecurity

- Use network data sets at Caida.org
  - Center for Applied Internet Data Analysis
  - Active Real-time Macroscopic Topology
- Test data sets:
  - Bars and Stripes model
  - Handwritten digits (MNIST)
  - KDD network traffic

Boltzmann Machines (BM)

Technical Approach: Boltzmann machines

- Restricted Boltzmann machines (RBM)
  - k-step contrastive divergence
  - parallel tempering
- More complicated graphs for BM
  - Chimera graph (as in D-Wave II)

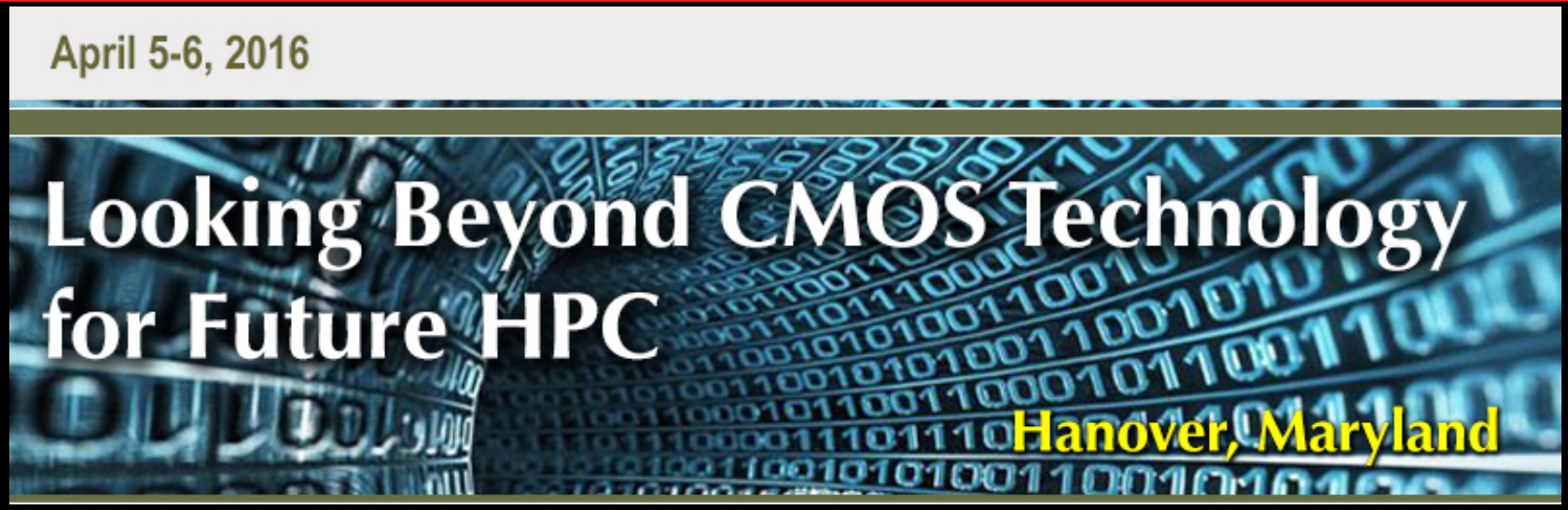


Boltzmann Machines (BM) with AQC

Technical Approach: Boltzmann machines

REDACTED SLIDE

Conclusions and Discussion

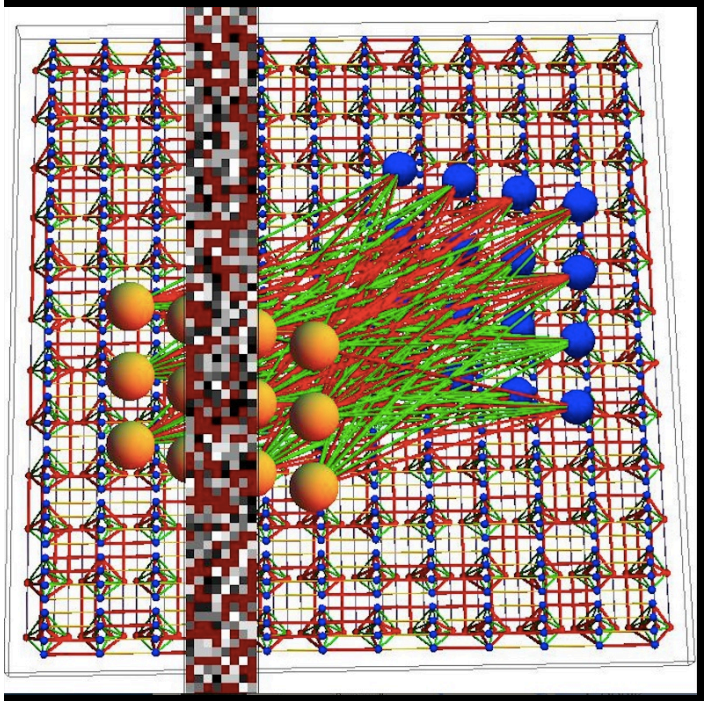


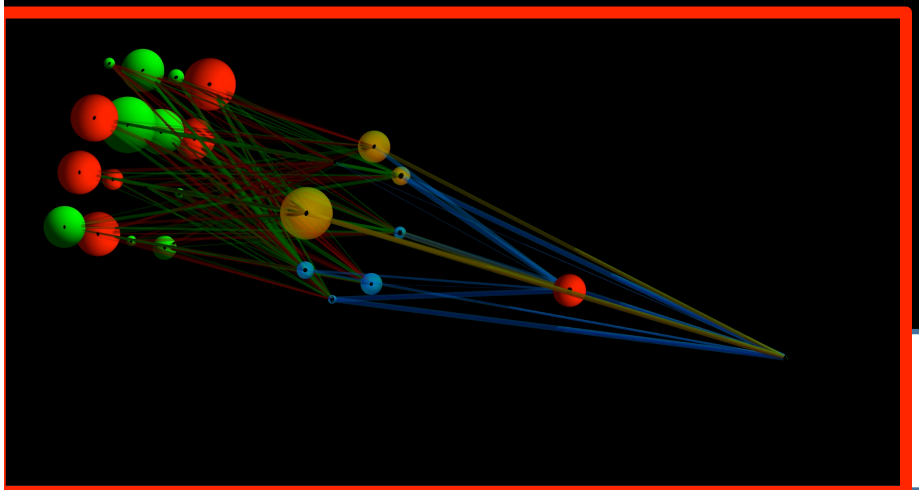
Look to Quantum

- Quantum dragons
- Quantum computing

Thank You!

AQC =
Adiabatic
Quantum
Computing





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